

Flavor Symmetry $L_\mu - L_\tau$ and quasi-degenerate Neutrinos

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Abstract

Current data implies three simple forms of the neutrino mass matrix, each corresponding to the conservation of a non-standard lepton charge. While models based on L_e and $L_e - L_\mu - L_\tau$ are well-known, little attention has been paid to $L_\mu - L_\tau$. A low energy mass matrix conserving $L_\mu - L_\tau$ implies quasi-degenerate light neutrinos. Moreover, it is μ - τ symmetric and therefore (in contrast to L_e and $L_e - L_\mu - L_\tau$) automatically predicts maximal atmospheric neutrino mixing and zero U_{e3} . A see-saw model based on $L_\mu - L_\tau$ is investigated and testable predictions for the neutrino mixing observables are given. Renormalization group running below and in between the see-saw scales is taken into account in our analysis, both numerically and analytically.

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1 Introduction

Impressive steps towards identifying the structure of the neutrino mass matrix m_ν have been taken in recent years [1]. In the charged lepton basis m_ν is diagonalized by the PMNS matrix [2] U defined via

$$U^T m_\nu U = \text{diag}(m_1, m_2, m_3) . \quad (1)$$

It can be parametrized as

$$U = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix} P , \quad (2)$$

where $P = \text{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1)$ contains the Majorana phases and c_{ij} , s_{ij} are defined as $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively.

The smallness of neutrino masses is commonly attributed to the see-saw mechanism [3], which states that integrating out three heavy Majorana neutrinos with masses $M_{1,2,3}$ leads to a low energy neutrino mass matrix given by

$$m_\nu = -m_D M_R^{-1} m_D^T . \quad (3)$$

Here M_R denotes the mass matrix of the three Majorana neutrinos $N_{1,2,3}$ and m_D is a Dirac mass matrix resulting from the coupling of the Higgs doublet to the left-handed neutrinos and the N_i .

On the other hand, the unexpected form of the mixing scheme implied by the data, namely one small (θ_{13}) and two large ($\theta_{12,23}$ with $\theta_{12} \lesssim \theta_{23}$) angles in combination with a rather moderate mass hierarchy, poses a more difficult problem. It might hint towards a particular symmetry, which seems not to be present in the quark sector. In order to find this underlying symmetry of the neutrino mass matrix, one might consider minimal models and/or symmetries which are able to accommodate the data. For instance, one could introduce a non-standard flavor charge corresponding to an Abelian $U(1)$ symmetry and demand that the neutrino mass matrix conserves this flavor charge. This will imprint a particular form onto m_ν and lead to certain mixing scenarios which can then be compared to the data. For three active flavor states there are the charges L_e , L_μ and L_τ , out of which one can construct 10 linearly independent combinations of the form $c_e L_e + c_\mu L_\mu + c_\tau L_\tau$ with $c_{e,\mu,\tau} = 0$ or ± 1 . Neutrino data implies that these symmetries must be approximate. Nevertheless, already at the present stage we are capable of ruling out six of them, leaving us with four testable cases. Those four cases are easily to distinguish, because their predictions for the neutrino mass spectrum and for the effective mass governing neutrinoless double beta decay [4] are very different:

- conservation of L_e implies the following form for the neutrino mass matrix [5]:

$$m_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix} , \quad (4)$$

where a, b, d are in general complex parameters of order one. Introducing small breaking terms in the first row of m_ν leads to a normal hierarchy, i.e., small neutrino masses $m_3 \gg m_2 > m_1$ and a very small effective mass of order $\sqrt{\Delta m_\odot^2} \simeq 10^{-3}$ eV, where Δm_\odot^2 denotes the mass squared difference of solar neutrinos. The common mass scale of the matrix (4) is the square root the mass squared difference of atmospheric neutrinos Δm_A^2 ;

- conservation of $L_e - L_\mu - L_\tau$ demands the following form of the mass matrix [6]:

$$m_\nu = \sqrt{\Delta m_A^2} \begin{pmatrix} 0 & a & b \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}. \quad (5)$$

The result is an inverted hierarchy, i.e., $m_2 \gtrsim m_1 \gg m_3$ and an effective mass of order $\sqrt{\Delta m_A^2} \simeq 0.04$ eV. Contributions from the charged lepton sector are crucial in order to reach accordance with the data [7], because solar neutrino mixing is predicted to be maximal by Eq. (5). Since the deviation from maximal neutrino mixing is of order of the Cabibbo angle [8], matrices conserving $L_e - L_\mu - L_\tau$ are ideal candidates for incorporating Quark–Lepton–Complementarity (QLC) [9];

- conservation of $L_\mu - L_\tau$ has caught only little attention. The mass matrix implied by this symmetry has the form [10, 11, 12]:

$$m_\nu = m_0 \begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}. \quad (6)$$

One is lead to quasi-degenerate light neutrinos, i.e., masses much larger than the mass splitting, and the effective mass is of order of the common mass scale $m_0 \lesssim$ eV.

The other possibilities, i.e., conservation of L_μ , L_τ , or $L_e - L_\mu$, $L_e - L_\tau$ or $L_e + L_\mu - L_\tau$, $L_e - L_\mu + L_\tau$ are ruled out phenomenologically. Finally, there is a tenth possibility corresponding to the conservation of the total lepton charge $L_e + L_\mu + L_\tau$. The consequence would be that neutrinos are Dirac particles and that there is no neutrinoless double beta decay. Without more theoretical input, other than just $L_e + L_\mu + L_\tau$, the neutrino mass spectrum and the mixing angles are arbitrary (for some recent models for Dirac neutrinos, see for instance [13]).

As already mentioned, the probably most puzzling aspect of neutrino mixing is that one angle is close to zero and another one close to maximal. This indicates the presence of a

μ - τ exchange symmetry [14], which results in a mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ \cdot & D & E \\ \cdot & \cdot & D \end{pmatrix}. \quad (7)$$

By comparing this matrix with the three matrices from Eqs. (4, 5, 6) we see that only $L_\mu - L_\tau$ automatically possesses a μ - τ symmetry. Therefore, $\theta_{23} = \pi/4$ and $U_{e3} = 0$ are generic predictions of mass matrices obeying the flavor symmetry $L_\mu - L_\tau$. Phenomenologically, $\tan \theta_{23}$ is a priori not equal to one for (broken) L_e and $L_e - L_\mu - L_\tau$, but just a ratio of two numbers.

On the more formal side, it turns out that the combination $L_\mu - L_\tau$ is anomaly free and can be gauged in the Standard Model (SM) [15, 16]. In fact, also the combination $L_e - L_\mu$ or $L_e - L_\tau$ could be chosen to be gauged. However, (as indicated above) these two combinations are phenomenologically ruled out, since they typically predict two small neutrino mixing angles [16].

Here we wish to analyze the consequences of a see-saw model based on the $U(1)$ flavor symmetry $L_\mu - L_\tau$. We will break the symmetry in a minimal way so that low energy neutrino data can be accommodated and that leptonic CP is possible. Interesting and testable phenomenology is predicted. Since neutrino physics has entered a precision era, a full study of any model has to take radiative corrections [17, 18, 19, 20] into account. In the framework of $U(1)$ models under consideration, one should note the following: models based on L_e receive only minor corrections due to RG effects because these are in general very small for the normal hierarchy. If $L_e - L_\mu - L_\tau$ is the symmetry from which a model originates, then the opposite CP parities of the leading mass states suppress the running, thus leading again to small effects. However, for quasi-degenerate neutrinos, thus in particular for $L_\mu - L_\tau$, one expects almost certainly large RG corrections. Moreover, a full study of RG effects in a see-saw model should not only consider the running from the lowest heavy neutrino mass M_1 down to the weak scale, but also the running between the different heavy neutrino scales M_3 to M_2 and M_2 to M_1 [21, 22, 23]. Those contributions can be at least as important as the usual ones from M_1 to M_Z . In our model we explicitly show that they are.

2 See-saw Model for $L_\mu - L_\tau$

Recently the case $L_\mu - L_\tau$ has been re-analyzed and a simple see-saw model has been constructed [12]. Apart from the Standard Model (SM) particle content there are only three extra SM singlets required. Those are heavy Majorana neutrinos which are used to generate light neutrino masses via the see-saw mechanism [3]. The charge assignment of the particles under $L_\mu - L_\tau$ is given in Table 1. As a consequence, the charged lepton mass

	$(\nu_e, e)_L$	$(\nu_\mu, \mu)_L$	$(\nu_\tau, \tau)_L$	N_1, e_R	N_2, μ_R	N_3, τ_R	Φ
$L_\mu - L_\tau$	0	1	-1	0	1	-1	0

Table 1: Particle content and charge under the $U(1)$ symmetry corresponding to $L_\mu - L_\tau$.

matrix is real and diagonal and the Lagrangian responsible for the neutrino masses reads

$$\begin{aligned}
-\mathcal{L} &= \bar{\nu}_\alpha (m_D)_{\alpha i} N_i + \frac{1}{2} N_i^T C^{-1} (M_R)_{ij} N_j + h.c. \\
&= \Phi (a \bar{\nu}_e N_1 + b \bar{\nu}_\mu N_2 + d \bar{\nu}_\tau N_3) + \frac{M}{2} (X N_1^T C^{-1} N_1 + Y N_2^T C^{-1} N_3) + h.c.
\end{aligned} \tag{8}$$

Here, M is the high mass scale of the heavy singlets, Φ the Higgs doublet with its vacuum expectation value $v/\sqrt{2} \simeq 174$ GeV and C is the charge conjugation matrix. In terms of mass matrices, we have

$$m_D = v \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & d \end{pmatrix} \text{ and } M_R = M \begin{pmatrix} X & 0 & 0 \\ \cdot & 0 & Y \\ \cdot & \cdot & 0 \end{pmatrix}. \tag{9}$$

At this stage, we are dealing with real parameters a, b, d, X, Y . After integrating out the heavy singlet states, the neutrino mass matrix $m_\nu = -m_D M_R^{-1} m_D^T$ is given by

$$m_\nu = -\frac{v^2}{M} \begin{pmatrix} \frac{a^2}{X} & 0 & 0 \\ \cdot & 0 & \frac{bd}{Y} \\ \cdot & \cdot & 0 \end{pmatrix}. \tag{10}$$

We need to break $L_\mu - L_\tau$ in order to generate successful phenomenology¹. Note that the mass matrix from above has maximal 23 mixing, arbitrary 12 and zero 13 mixing as well as only one non-vanishing Δm^2 , given by $v^4/M^2 (b^2 d^2/Y^2 - a^4/X^2)$. This is the mass splitting associated with the solar neutrino oscillations. We consider from now on the situation in which the symmetry is softly broken by additional small parameters in M_R . Moreover, in contrast to Ref. [12], we allow for CP violation. Since the Dirac and charged lepton mass matrices are already diagonal, we can eliminate the possible phases of a, b, d by redefining the lepton doublets. Consequently, prior to breaking only two phases are present, which we assign to the heavy singlets such that we replace in Eq. (9) X with $X e^{i\phi}$ and Y with $Y e^{i\omega}$. The first and minimal approach is to add just one small entry to M_R . For instance, we can add to the 12 element an entry $\epsilon e^{i\chi}$ with real $\epsilon \ll 1$. As a result, the low energy

¹For alternative breaking scenarios in case of CP conservation, see [12].

mass matrix will read

$$m_\nu = \frac{v^2}{M} \begin{pmatrix} \frac{a^2 e^{-i\phi}}{X} & 0 & \frac{-a d \epsilon e^{i(\chi-\omega-\phi)}}{X Y} \\ \cdot & 0 & \frac{b d e^{-i\omega}}{Y} \\ \cdot & \cdot & \frac{d^2 \epsilon^2 e^{i(2\chi-2\omega-\phi)}}{X Y^2} \end{pmatrix}. \quad (11)$$

It is interesting to note that there is no CP violation in oscillation experiments or at high energy. This can be shown, for instance, by rephasing the elements of Eq. (11) or by considering basis invariant quantities. Let us define here the quantity J_{CP} , to which any CP violating effect in neutrino oscillation experiments will be proportional. In terms of mixing angles and the Dirac phase δ it is given by

$$J_{CP} = \text{Im}\{U_{e1} U_{\mu 2} U_{e2}^* U_{\mu 1}^*\} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta. \quad (12)$$

From the mass matrix m_ν , in turn, one can deduce [24]

$$\text{Im}\{h_{12} h_{23} h_{31}\} = \Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2 J_{CP}, \text{ where } h = m_\nu^\dagger m_\nu. \quad (13)$$

Using the mass matrix from Eq. (11) to evaluate this expression yields that J_{CP} is zero. In order to have CP violation, we are therefore forced to add another perturbation to M_R :

$$M_R = M \begin{pmatrix} X e^{i\phi} & \epsilon_1 e^{i\psi_1} & 0 \\ \cdot & \epsilon_2 e^{i\psi_2} & Y e^{i\omega} \\ \cdot & \cdot & 0 \end{pmatrix}. \quad (14)$$

However, one can show that there is only one physical phase, and it therefore suffices to consider

$$M_R = M \begin{pmatrix} X & \epsilon_1 & 0 \\ \cdot & \epsilon_2 e^{i\varphi} & Y \\ \cdot & \cdot & 0 \end{pmatrix} \quad (15)$$

to obtain the following low energy effective mass matrix

$$m_\nu = -\frac{v^2}{M} \begin{pmatrix} \frac{a^2}{X} & 0 & \frac{-a d \epsilon_1}{X Y} \\ \cdot & 0 & \frac{b d}{Y} \\ \cdot & \cdot & \frac{-d^2 (X \epsilon_2 e^{i\varphi} - \epsilon_1^2)}{X Y^2} \end{pmatrix}. \quad (16)$$

For the rest of this contribution we will focus on this mass matrix. Let us therefore analyze in detail the implied phenomenology.

A matrix with zero entries in the $e\mu$ and $\mu\mu$ elements has well-known predictions [25, 26]. In particular, only quasi-degenerate light neutrinos are compatible with such a matrix

and in addition it is required that the ee and the $\mu\tau$ elements are leading and of similar magnitude [26]. This is however just the approximate form of a mass matrix conserving $L_\mu - L_\tau$.

By using the definition of the mass matrix from Eq. (1) and inserting the conditions $m_{e\mu} = 0$ and $m_{\mu\mu} = 0$, one can obtain an expression for the ratio of the neutrino masses. Expanding in terms of the small parameter $|U_{e3}|$, one finds:

$$\begin{aligned} \left| \frac{m_1}{m_3} \right| &\simeq \tan^2 \theta_{23} - |U_{e3}| \cos \delta \cot \theta_{12} \frac{\tan \theta_{23}}{\cos^2 \theta_{23}} , \\ \left| \frac{m_2}{m_3} \right| &\simeq \tan^2 \theta_{23} + |U_{e3}| \cos \delta \tan \theta_{12} \frac{\tan \theta_{23}}{\cos^2 \theta_{23}} . \end{aligned} \quad (17)$$

Since θ_{23} is close to maximal and θ_{13} is small, the light neutrinos are obviously quasi-degenerate. From the ratio of masses one can calculate the ratio of the solar mass squared difference over the atmospheric one:

$$R \equiv \frac{\Delta m_\odot^2}{\Delta m_A^2} = \frac{m_2^2 - m_1^2}{|m_3^2 - m_2^2|} \simeq \left| 4 |U_{e3}| \cos \delta \frac{\tan \theta_{23}}{\cos 2\theta_{23}} \frac{\sin^2 \theta_{23}}{\sin 2\theta_{12}} \right| . \quad (18)$$

As R is inverse proportional to the rather small quantity $\cos 2\theta_{23}$, it is necessary that $\sin \theta_{13} \cos \delta = \text{Re } U_{e3} \ll 1$. Hence, the Dirac phase should be located around its maximal value $\pi/2$, i.e., CP violation is close to maximal. The larger $\sin \theta_{13}$ is, the smaller $\cos \delta$ has to be, which indicates that for sizable $|U_{e3}|$ only maximal CP violation will occur. Furthermore, to keep R small, the angle θ_{23} can not become maximal. If it is located above $\pi/4$, then it follows from Eq. (17) that $m_1 > m_3$ and $m_2 > m_3$, which leads to an inverted ordering. Equivalently, for $\theta_{23} < \pi/4$ one has $m_3 > m_1$ and $m_3 > m_2$ and the mass ordering is normal. These aspects of the phenomenology are almost independent on the precise value of θ_{12} , which — as we will see later on — can receive large renormalization corrections. The effective mass governing neutrinoless double beta decay can be written as

$$\langle m \rangle = \frac{v^2}{M} \frac{a^2}{X} \simeq \tan^2 \theta_{23} \sqrt{\frac{\Delta m_A^2}{|1 - \tan^4 \theta_{23}|}} , \quad (19)$$

which shows again that maximal atmospheric neutrino mixing is forbidden. For later use, we note here that for the neutrino texture under discussion typically $\varphi_1 \simeq \varphi_2 \simeq \pi$ holds, i.e., the two Majorana phases have very similar values [26]. This will be very useful when we analyze the renormalization aspects of the model.

Let us now take a closer look at the structure of the Dirac and heavy Majorana mass matrices. Since the charged leptons display a hierarchy, it is natural to assume that also the eigenvalues of the Dirac mass matrix are hierarchical. Then it is required that also the heavy Majorana neutrinos display a hierarchy in the form of $Y \gg X$. Typical values of the parameters which in this case successfully reproduce the neutrino data are $Y = \mathcal{O}(1)$, $a = \mathcal{O}(0.01)$, $b \sim d = \mathcal{O}(0.1)$, $X = \mathcal{O}(0.001)$, $\epsilon_1 = \mathcal{O}(0.001)$ and $\epsilon_2 = \mathcal{O}(0.1)$. With these values, the eigenvalues of M_R are approximatively given by $M X$ and $M (Y \pm \epsilon_2/2)$. We plot in Fig. 1 some of the correlations obtained for parameters around the values given

above. The oscillation parameters are required to lie within the 3σ ranges from Ref. [27]. In fact, just demanding zero entries in the $e\mu$ and $\mu\mu$ elements will lead to such a behavior of the observables [26]. Indeed, as can be seen from the Figure, the neutrinos are close in mass, atmospheric neutrino mixing is not maximal and lies above (below) $\pi/4$ for the inverted (normal) mass ordering. Furthermore, CP violation is basically maximal for sizable values of $|U_{e3}|$.

3 Renormalization Group Effects

As the flavor symmetry $L_\mu - L_\tau$ leads to quasi-degenerate neutrino masses, strong running of the mixing angles is generically expected [18, 20]. Typically, the running of the mixing angles θ_{ij} in a quasi-degenerate mass scheme with a common mass scale m_0 is proportional to $m_0^2/(m_i^2 - m_j^2)$ and therefore particularly strong for θ_{12} . This behavior turns out to hold not only for the running between the scale of the lightest heavy Majorana neutrino and the weak scale, but also when the running between the see-saw scales is taken into account [21, 22]. The heavy Majorana neutrinos have to be integrated out one after the other, leading to a series of effective theories. It turns out that the running of the observables between the scales M_3 and M_2 or M_2 and M_1 can be of the same order as the running between M_1 and M_Z . In general this leads to quite involved expressions for the β -functions. However, in our case the structure of the Dirac and charged lepton mass matrices (i.e., the fact that they are diagonal) simplifies matters considerably and allows for some analytic understanding of the numerical results. All of the expressions we give in this Section are understood to be precise to order $|U_{e3}|$ and can be obtained with the help of Refs. [20, 22]. For instance, the β -functions for the mixing angles are:

$$\begin{aligned} 16\pi^2 \dot{\theta}_{12} &\simeq \frac{m_0^2}{\Delta m_\odot^2} (1 + \cos(\varphi_2 - \varphi_1)) \sin 2\theta_{12} [P_{11} - (P_{22} \cos^2 \theta_{23} + P_{33} \sin^2 \theta_{23})] , \\ 32\pi^2 \dot{\theta}_{13} &\simeq \frac{m_0^2}{\Delta m_A^2} (\cos(\delta - \varphi_1) - \cos(\delta - \varphi_2)) \sin 2\theta_{12} \sin 2\theta_{23} (P_{22} - P_{33}) , \\ 16\pi^2 \dot{\theta}_{23} &\simeq \frac{m_0^2}{\Delta m_A^2} [(1 + \cos \varphi_2) \cos^2 \theta_{12} + (1 + \cos \varphi_1) \sin^2 \theta_{12}] \sin 2\theta_{23} (P_{22} - P_{33}) . \end{aligned} \quad (20)$$

Note that since $\varphi_1 \simeq \varphi_2$ there is no cancellation in the first relation for $\dot{\theta}_{12}$. The matrix P in the above equations is given in the SM and in the Minimal Supersymmetric Standard Model (MSSM) by

$$P = \begin{cases} -\frac{3}{2}Y_e Y_e^\dagger + \frac{1}{2}Y_\nu Y_\nu^\dagger & \text{SM} , \\ Y_e Y_e^\dagger + Y_\nu Y_\nu^\dagger & \text{MSSM} . \end{cases} \quad (21)$$

Since in our model both $Y_\nu = m_D/v$ and $Y_e = m_\ell/v$ are diagonal, P is diagonal as well and no off-diagonal elements appear in Eq. (20). With the values of the parameters in m_D as given in Section 2 (i.e., $a \ll b \sim d = \mathcal{O}(0.1)$) we can safely neglect P_{11} . This

leads in particular to a negative β -function for θ_{12} . Hence, this angle will become larger when evolved from high to low scales. Neglecting further the charged lepton Yukawas in Y_e above the see-saw scales and noting that $P_{22} \simeq b^2/2$ and $P_{33} \simeq d^2/2$ for the SM and twice those values for the MSSM, we see that the running of θ_{13} and θ_{23} is suppressed with respect to the running of θ_{12} due to two reasons: firstly, it is inverse proportional to Δm_A^2 and secondly, it is proportional to $P_{22} - P_{33} \propto b^2 - d^2$, which is smaller than $P_{22} + P_{33}$, on which the running of θ_{12} (approximately) depends. Hence, the running of θ_{13} and θ_{23} is suppressed by $R(b^2 - d^2)/b^2$ above the see-saw scales.

Below the see-saw scales only the tau-lepton Yukawa coupling y_τ governs the RG corrections. In this regime the evolution goes like [20]

$$16\pi^2 \dot{\theta}_{12} \simeq -y_\tau^2 \sin 2\theta_{12} \sin^2 \theta_{23} \frac{m_0^2}{\Delta m_\odot^2} (1 + \cos(\varphi_2 - \varphi_1)) , \quad (22)$$

in the MSSM, which is again negative and leads to sizable running. The formulae for the running of $\theta_{13,23}$ are again suppressed by roughly a factor R .

The phases stay almost constant in the whole range, because one can show that for our model $\dot{\varphi}_1$ and $\dot{\varphi}_2$ are mainly proportional to $\sin \varphi_1$ and $\sin \varphi_2$, respectively. Roughly the same behavior is found below M_1 . Finally, the RG effects on the neutrino masses correspond predominantly to a rescaling, since the flavor-diagonal couplings, i.e., gauge couplings and the quartic Higgs coupling, dominate the evolution [19].

We can analyze if the zero entries in the mass matrix Eq. (16) remain zero entries. Below the see-saw scale it is well known that the RG corrections are multiplicative on the mass matrix, a fact which leaves zero entries zero. Taking the running in between the heavy Majorana masses into account, one can factorize the renormalization group effects Z_{ext} from the tree-level neutrino mass matrix in the MSSM as [23]

$$m_\nu = Z_{\text{ext}}^T m_\nu^0 Z_{\text{ext}} . \quad (23)$$

Since in our model the neutrino and the charged lepton Yukawa coupling matrices are diagonal, the renormalization group effects are flavour diagonal, too. Therefore, texture zeroes in the charged lepton basis remain zero. With the already mentioned simplifications, Z_{ext} is approximately² given by

$$16\pi^2 (Z_{\text{ext}}^{\text{MSSM}} - \mathbb{1}) = \text{diag} \left(0, b^2 \ln \frac{MY}{\Lambda}, d^2 \ln \frac{MY}{\Lambda} + y_\tau^2 \ln \frac{v}{\Lambda} \right) \\ + b^2 \ln \frac{MY}{\Lambda} + d^2 \ln \frac{MY}{\Lambda} + \left[-\frac{3}{5}g_1^2 - 3g_2^2 + 3y_t^2 \right] \ln \frac{v}{\Lambda} , \quad (24)$$

where (g_2, g_1) are the gauge couplings³ of the electroweak interactions $SU(2)_L \times U(1)_Y$, y_t and y_τ are the top quark and τ Yukawa couplings, respectively and Λ is the GUT scale of $2 \cdot 10^{16}$ GeV.

²As the perturbations are small, the mass eigenvalues of the heavy right-handed neutrinos are approximately given by $(M_1, M_2, M_3) \simeq M(X, Y(1 - \epsilon_2/2), Y(1 + \epsilon_2/2)) \simeq (X, Y, Y)$.

³We use GUT charge normalization for g_1 , i.e., $g_1^{\text{SM}} = \sqrt{\frac{3}{5}}g_1$.

L'	matrix	comments
L_e normal hierarchy always small RG effects	$\begin{pmatrix} 0 & 0 & 0 \\ \cdot & a & b \\ \cdot & \cdot & d \end{pmatrix}$	$R = \frac{\Delta m_{\odot}^2}{\Delta m_{\text{A}}^2} \simeq U_{e3} ^2$ $\tan^2 \theta_{\text{atm}} \simeq 1 + U_{e3} \simeq 1 + \sqrt{R}$ $\langle m \rangle \simeq \sqrt{\Delta m_{\text{A}}^2} U_{e3} ^2 \simeq \sqrt{\Delta m_{\odot}^2}$
$L_e - L_{\mu} - L_{\tau}$ inverted hierarchy small RG effects (pseudo-Dirac)	$\begin{pmatrix} 0 & a & b \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix}$	requires U_{ℓ} : ideal for QLC $\tan^2 \theta_{\odot} \simeq 1 - 4 U_{e3} \stackrel{?}{\simeq} 1 - 2\sqrt{2} \sin \theta_{\text{C}}$ $\langle m \rangle \simeq \sqrt{\Delta m_{\text{A}}^2}$
$L_{\mu} - L_{\tau}$ quasi-degeneracy always large RG effects	$\begin{pmatrix} a & 0 & 0 \\ \cdot & 0 & b \\ \cdot & \cdot & 0 \end{pmatrix}$	in leading order: $U_{e3} = 0$ and $\theta_{23} = \pi/4$ $\langle m \rangle \simeq m_0$

Table 2: The three simple $U(1)$ charges as implied by current data, the zeroth order mass matrix and some comments on their phenomenology.

In the extended SM, however, there are additional corrections which can not be factorized. They are responsible for the instability of texture zeroes under the renormalization group in the SM [22, 23]. We checked that for most observables the running behavior in the SM is similar to the running in case of the MSSM. The solar neutrino mixing angle receives however more RG corrections in the SM, a fact which can be traced back to the appearance of an $e\mu$ entry in the mass matrix Eq. (16). One might wonder at this point if this filling of zero entries would allow one to generate successful phenomenology from a mass matrix obeying the flavor symmetry $L_{\mu} - L_{\tau}$ without any breaking, i.e., just from Eq. (6). Recall, however, that in the SM $L_{\mu} - L_{\tau}$ is anomaly free and therefore the texture of the mass matrix is stable.

We plot in Fig. 2 the running of the angles, phases and masses for a typical example in the MSSM for $\tan \beta = 10$ [28]. The resulting neutrino parameters at the GUT scale are $\sin^2 \theta_{12} = 0.123$, $U_{e3} = 0.0484 e^{i4.73}$, $\sin^2 \theta_{23} = 0.481$, $(m_1, m_2, m_3) = (0.1527, 0.1533, 0.1653)$ eV with $\Delta m_{\odot}^2 = 1.9 \cdot 10^{-4}$ eV² and $\Delta m_{\text{A}}^2 = 3.8 \cdot 10^{-3}$ eV². They are changed by the renormalization group evolution to $\sin^2 \theta_{12} = 0.303$, $U_{e3} = 0.0496 e^{i4.73}$, $\sin^2 \theta_{23} = 0.481$, $(m_1, m_2, m_3) = (0.1152, 0.1155, 0.1245)$ eV with $\Delta m_{\odot}^2 = 7.9 \cdot 10^{-5}$ eV² and $\Delta m_{\text{A}}^2 = 2.1 \cdot 10^{-3}$ eV². Note that the phases and $\theta_{13,23}$ remain practically constant, whereas $\sin^2 \theta_{12}$ and $\Delta m_{\odot, \text{A}}^2$ are changed by factors of up to three, and that the running in between and above the see-saw scales is at least as important as the running below them. This implies that radiative corrections, in particular for θ_{12} and the mass squared differences, can be crucial if one wishes to make meaningful phenomenological predictions of a given model.

4 Conclusions and Summary

Identifying the structure of the mass matrix m_ν is one of the main goals of neutrino physics. Current data seems to point towards three interesting forms of m_ν , each corresponding to the (approximative) conservation of a non-standard lepton charge. While L_e and $L_e - L_\mu - L_\tau$ are well known cases — leading to the normal and inverted hierarchy, respectively — the third possibility $L_\mu - L_\tau$ has not been examined in as much detail. Since quasi-degenerate neutrinos are predicted by the conservation of $L_\mu - L_\tau$, this case is easily testable. Moreover, since the mass matrix is automatically μ - τ symmetric, further phenomenological support is provided. We stress again that the three possibilities L_e , $L_e - L_\mu - L_\tau$ and $L_\mu - L_\tau$ differ in particular by their prediction for neutrinoless double beta decay, a fact which will easily allow to identify the correct one. In Table 2 the relevant matrices and some aspects of their phenomenology are summarized. Here we analyzed the consequences of a see-saw model based on $L_\mu - L_\tau$, which predicts, e.g., non-maximal atmospheric neutrino mixing, large CP violation and quasi-degenerate neutrinos. This last aspect makes radiative corrections be particularly important and we carefully analyzed their effects both numerically and analytically. We showed explicitly that running between the see-saw scales is at least as important as running from the lowest see-saw scale down to low energies.

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References

- [1] For an overview of the current situation see, for instance: R. N. Mohapatra *et al.*, hep-ph/0412099; S. T. Petcov, Talk given at the 21st International Conference on Neutrino Physics and Astrophysics (Neutrino 2004), June 14 – 19, 2004, Paris, France, Nucl. Phys. Proc. Suppl. **143**, 159 (2005); hep-ph/0412410.
- [2] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957) and **34**, 247 (1958); Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [3] P. Minkowski, Phys. Lett. **B67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in *Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. 707; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

- [4] For recent overviews about the predictions for neutrinoless double beta decay, see S. Pascoli, S. T. Petcov and T. Schwetz, hep-ph/0505226; S. Choubey and W. Rodejohann, hep-ph/0506102.
- [5] W. Buchmuller and T. Yanagida, Phys. Lett. B **445**, 399 (1999); F. Vissani, JHEP **9811**, 025 (1998); see also the second paper in [6].
- [6] S. T. Petcov, Phys. Lett. B **110**, 245 (1982); an incomplete list of more recent studies is: R. Barbieri *et al.*, JHEP **9812**, 017 (1998); A. S. Joshipura and S. D. Rindani, Eur. Phys. J. C **14**, 85 (2000); R. N. Mohapatra, A. Perez-Lorenzana and C. A. de Sousa Pires, Phys. Lett. B **474**, 355 (2000); Q. Shafi and Z. Tavartkiladze, Phys. Lett. B **482**, 145 (2000). L. Lavoura, Phys. Rev. D **62**, 093011 (2000); T. Kitabayashi and M. Yasue, Phys. Rev. D **63**, 095002 (2001); K. S. Babu and R. N. Mohapatra, Phys. Lett. B **532**, 77 (2002); H. J. He, D. A. Dicus and J. N. Ng, Phys. Lett. B **536**, 83 (2002) H. S. Goh, R. N. Mohapatra and S. P. Ng, Phys. Lett. B **542**, 116 (2002); G. K. Leontaris, J. Rizos and A. Psallidas, Phys. Lett. B **597**, 182 (2004); W. Grimus and L. Lavoura, J. Phys. G **31**, 683 (2005).
- [7] P. H. Frampton, S. T. Petcov and W. Rodejohann, Nucl. Phys. B **687**, 31 (2004); S. T. Petcov and W. Rodejohann, Phys. Rev. D **71**, 073002 (2005).
- [8] W. Rodejohann, Phys. Rev. D **69**, 033005 (2004).
- [9] M. Raidal, Phys. Rev. Lett. **93**, 161801 (2004); H. Minakata and A. Y. Smirnov, Phys. Rev. D **70**, 073009 (2004); for a review and more references, see H. Minakata, hep-ph/0505262.
- [10] P. Binetruiy *et al.*, Nucl. Phys. B **496**, 3 (1997); N. F. Bell and R. R. Volkas, Phys. Rev. D **63**, 013006 (2001).
- [11] K. S. Babu, E. Ma and J. W. F. Valle, Phys. Lett. B **552**, 207 (2003); M. Hirsch *et al.*, Phys. Rev. D **69**, 093006 (2004).
- [12] S. Choubey and W. Rodejohann, Eur. Phys. J. C **40**, 259 (2005).
- [13] N. Arkani-Hamed *et al.*, Phys. Rev. D **64**, 115011 (2001); F. Borzumati and Y. Nomura, Phys. Rev. D **64**, 053005 (2001); R. Kitano, Phys. Lett. B **539**, 102 (2002); R. Arnowitt, B. Dutta and B. Hu, Nucl. Phys. B **682**, 347 (2004); S. Abel, A. Dedes and K. Tamvakis, Phys. Rev. D **71**, 033003 (2005); C. Hagedorn and W. Rodejohann, JHEP **0507**, 034 (2005); S. Antusch, O. J. Eyton-Williams and S. F. King, hep-ph/0505140.
- [14] C. S. Lam, Phys. Lett. B **507**, 214 (2001); K. R. S. Balaji, W. Grimus and T. Schwetz, Phys. Lett. B **508**, 301 (2001); W. Grimus and L. Lavoura, JHEP **0107**, 045 (2001); Eur. Phys. J. C **28**, 123 (2003); E. Ma, Phys. Rev. D **66**, 117301 (2002); T. Kitabayashi and M. Yasue, Phys. Rev. D **67**, 015006 (2003); P. F. Harrison and

- W. G. Scott, Phys. Lett. B **547**, 219 (2002); Y. Koide, Phys. Rev. D **69**, 093001 (2004); W. Grimus *et al.*, Nucl. Phys. B **713**, 151 (2005); R. N. Mohapatra, JHEP **0410**, 027 (2004); R. N. Mohapatra, S. Nasri and H. B. Yu, hep-ph/0502026; C. S. Lam, hep-ph/0503159; I. Aizawa, T. Kitabayashi and M. Yasue, hep-ph/0504172; K. Matsuda and H. Nishiura, hep-ph/0506192.
- [15] X. G. He *et al.*, Phys. Rev. D **43**, 22 (1991); Phys. Rev. D **44**, 2118 (1991); see also E. Ma, D. P. Roy and S. Roy, Phys. Lett. B **525**, 101 (2002).
- [16] R. Foot, hep-ph/0505154.
- [17] K. S. Babu, C. N. Leung and J. T. Pantaleone, Phys. Lett. B **319**, 191 (1993); S. Antusch *et al.*, Phys. Lett. B **519**, 238 (2001).
- [18] M. Tanimoto, Phys. Lett. B **360**, 41 (1995); J. R. Ellis and S. Lola, Phys. Lett. B **458**, 310 (1999). P. H. Chankowski, W. Krolikowski and S. Pokorski, Phys. Lett. B **473**, 109 (2000); J. A. Casas *et al.*, Nucl. Phys. B **573**, 652 (2000); N. Haba, Y. Matsui and N. Okamura, Eur. Phys. J. C **17**, 513 (2000); K. R. S. Balaji *et al.*, Phys. Rev. D **63**, 113002 (2001); M. Frigerio and A. Y. Smirnov, JHEP **0302**, 004 (2003).
- [19] P. H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A **17**, 575 (2002).
- [20] S. Antusch *et al.*, Nucl. Phys. B **674**, 401 (2003).
- [21] S. F. King and N. N. Singh, Nucl. Phys. **B591** 3, (2000); S. Antusch *et al.*, Phys. Lett. **B538** 87, (2002); B. Stech and Z. Tavartkiladze, Phys. Rev. **D70** 035002, (2004); J. W. Mei, Phys. Rev. D **71**, 073012 (2005); J. Ellis *et al.*, hep-ph/0506122.
- [22] S. Antusch *et al.*, JHEP **0503**, 024 (2005).
- [23] M. Lindner, M. A. Schmidt and A. Y. Smirnov, hep-ph/0505067.
- [24] G. C. Branco *et al.*, Phys. Rev. D **67**, 073025 (2003).
- [25] P. H. Frampton, S. L. Glashow and D. Marfatia, Phys. Lett. B **536**, 79 (2002); Z. Z. Xing, Phys. Lett. B **539**, 85 (2002) B. R. Desai, D. P. Roy and A. R. Vaucher, Mod. Phys. Lett. A **18**, 1355 (2003); W. L. Guo and Z. Z. Xing, Phys. Rev. D **67**, 053002 (2003).
- [26] Z. Z. Xing, Phys. Lett. B **530**, 159 (2002).
- [27] M. Maltoni *et al.*, New J. Phys. **6**, 122 (2004); hep-ph/0405172v4.
- [28] Such figures can be obtained with the software package REAP introduced in [22] and available at <http://www1.physik.tu-muenchen.de/~rge/>.

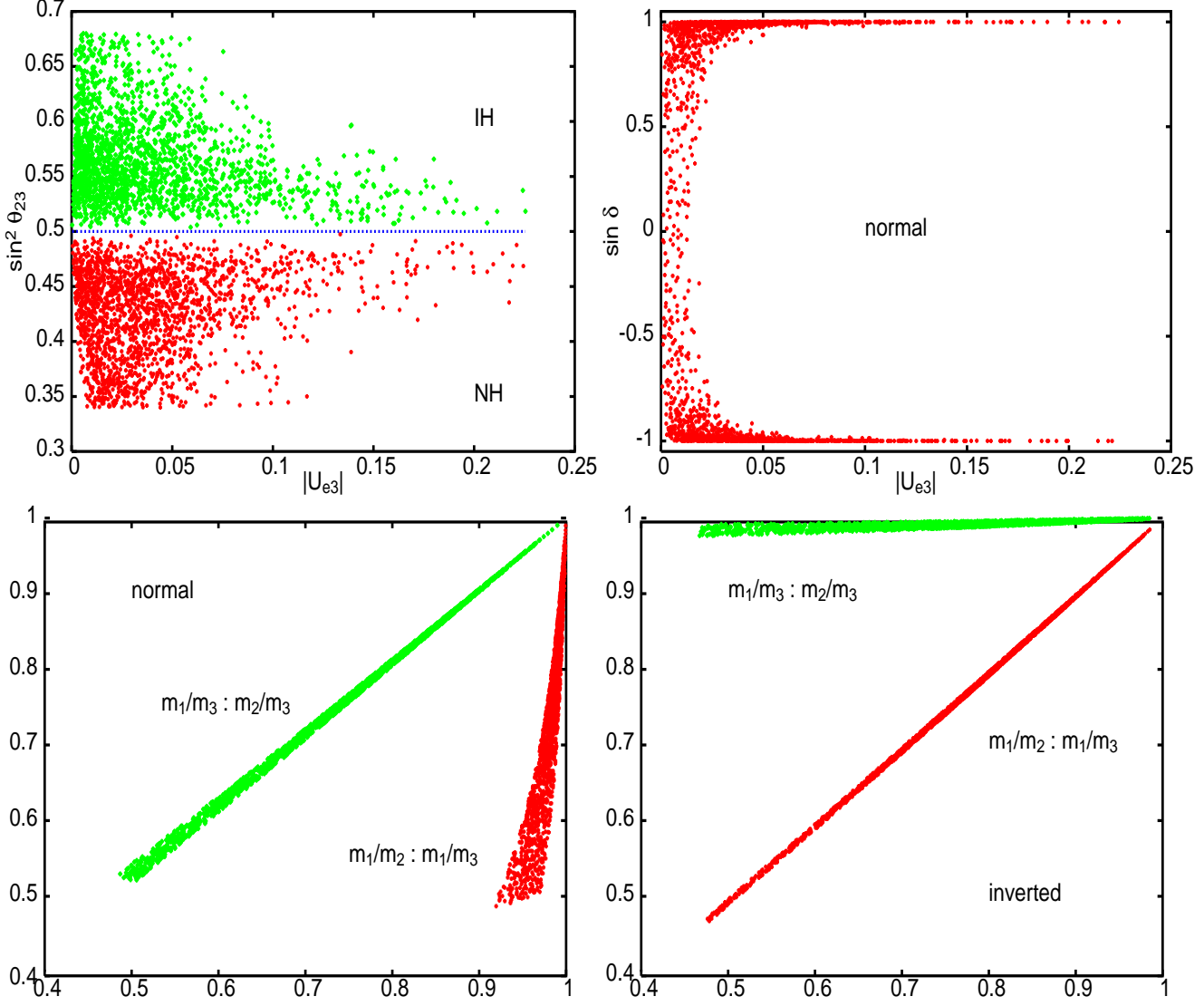


Figure 1: Scatter plots of some of the observables resulting from a mass matrix with zero $e\mu$ and $\mu\mu$ elements. We required the 3σ values for the oscillation parameters from [27].

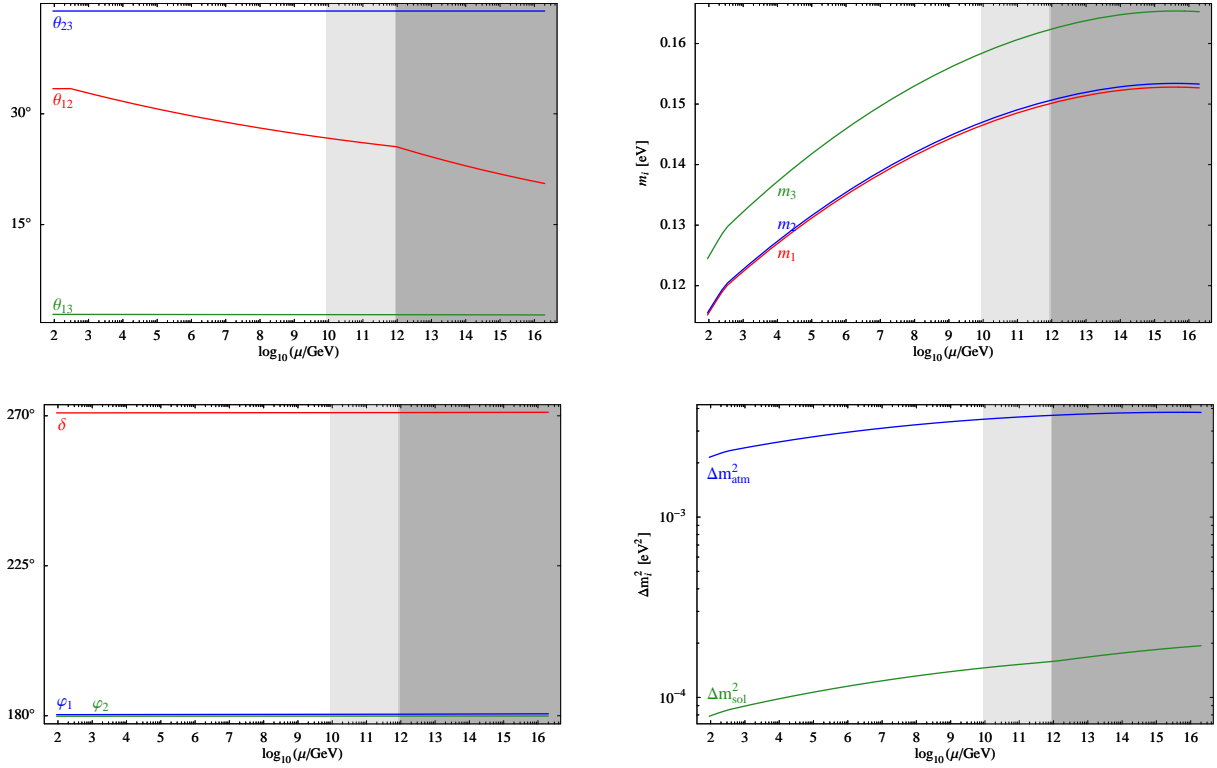


Figure 2: Typical plot for the evolution of the mixing angles and masses from the GUT scale 2×10^{16} GeV to low scale. In this example, we have $\tan \beta = 10$ and the parameters in the matrix Eq. (16) at the GUT scale are $a = 0.0066926$, $b = 0.0692883$, $c = 0.0697464$, $X = 0.0096528$, $Y = 1$, $\epsilon_1 = 0.0005595$, $\epsilon_2 = 0.0749098$, $\varphi = 2.45376$ and $M = 9.098937108 \cdot 10^{11}$ GeV.